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Expected Residual Vibration of Traditional and Hybrid Input Shaping Designs

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Introduction

IN the modeling of flexible structures, there is generally uncertainty in both the frequencies and damping constants; thus, it is important that the control methods for such systems be robust to these uncertainties. Input shaping has been successfully applied for controlling flexible structures while being insensitive to modeling errors.[1,2] However, previous studies have primarily used a measure of insensitivity that does not incorporate information about the probability distribution of parameter variations. The probability distribution of parameters can often be approximated in particular applications. For instance, in mass manufactured disk-drive systems, variations in the read/write arm mass and length can be mapped to determine the expected variations in the natural frequency. Similarly, operating temperature changes can affect the viscosity of coatings and alter the damping coefficients of the system. In such applications, data can be taken to estimate the distribution of parameters.

We investigate a recently proposed input shaping method[3] that suggests a measure of performance in terms of the expected level of residual vibration and easily incorporates the probability distribution of parameter variations. We also propose a very straightforward and computationally efficient technique to derive shapers that yield near-optimal performance. An analysis is presented for several types of optimal and near-optimal input shapers, and some interesting trends and qualitative behaviors of the shapers are shown.

Expected Residual Vibration of Shaper Designs

For simplicity of exposition, we limit the discussion to one-bending-mode flexible structures. The residual vibration resulting from a sequence of impulses is[1]

$$V(\omega,\zeta,t_v) = e^{-\zeta\omega t_v}\left\{\left[\sum_{i=1}^{m} A_i e^{\zeta\omega t_i}\cos(\omega_d t_i)\right]^2 + \left[\sum_{i=1}^{m} A_i e^{\zeta\omega t_i}\sin(\omega_d t_i)\right]^2\right\}^{\frac{1}{2}} \tag{1}$$

where $\omega_d = \omega\sqrt{(1-\zeta^2)}$; there are m impulses, $i = 1, \ldots, m$; A_i is the amplitude of the ith impulse; t_i is the time location of the ith impulse; and $t_v \geq t_m$ is the time (after the end of the shaped input command) at which the residual vibration is computed.

Constraints to account for actuator limits are also included. Requiring that the impulse amplitudes are positive and sum to unity

$$A_i \geq 0, \qquad \sum_{i=1}^{m} A_i = 1, \qquad i = 1,\ldots,m \tag{2}$$

guarantees that 1) the final set point of the system is the same for shaped commands as for unshaped commands and 2) the shaper can be used with any unshaped input without violating the actuator limits if the original unshaped command does not violate them.[1]

In practice, we may have some knowledge of the statistical nature of plant parameter variation, and it may be useful to incorporate this knowledge into a performance measure J that evaluates the expected level of residual vibration

$$J = \int_0^1 \int_0^\infty V(\omega,\zeta,t_v)\, f(\zeta,\omega)\,\mathrm{d}\omega\,\mathrm{d}\zeta \tag{3}$$

where f is a joint probability density function of the actual system frequency and damping. This performance measure has several advantages over the traditional insensitivity[1,2,4] measure: The robustness with respect to damping is taken into account, the frequency and damping intervals of concern are selectable, and these intervals can be weighted (with a probability density function).

A simpler performance index such as

$$J = \int_0^\infty V(\omega,\zeta,t_v)\, f(\omega)\,\mathrm{d}\omega \tag{4}$$

can be used if the damping variation is expected to be small. Similarly, the performance index

$$J = \int_0^1 V(\omega,\zeta,t_v)\, f(\zeta)\,\mathrm{d}\zeta \tag{5}$$

can be used if we are concerned primarily with damping uncertainty. In this Note, we consider one-mode systems, but additional modes can be easily incorporated by extending the optimization criterion to minimizing $\Sigma_i J_i$, $i = 1,\ldots,n$, where each J_i is the expected level of residual vibration (3), (4), or (5) due to each flexible mode.

We consider uniform and Gaussian distributions for parameter variation. For the performance indices (4) and (5) that only include variations in one parameter, with uniform distribution, the parameter p, which is either the natural frequency ω or the damping coefficient ζ, has the probability density function

$$f(p) = \begin{cases} \dfrac{1}{p_{\mathrm{hi}} - p_{\mathrm{lo}}}, & p \in [p_{\mathrm{lo}}, p_{\mathrm{hi}}] \\ 0, & \text{otherwise} \end{cases} \tag{6}$$

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with Gaussian distribution, the parameter p (ω or ζ) has the following probability density function:

$$f(p) = \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left[-\frac{(p-p_0)^2}{2\sigma^2}\right], \qquad p \in \boldsymbol{R} \tag{7}$$

where p_0 is the modeled parameter value.

For the performance index (3), where variations in both the natural frequency and damping are included, we consider the following distributions.

Uniform:

$$f(\zeta,\omega) = \begin{cases} \dfrac{1}{(\zeta_{\mathrm{hi}}-\zeta_{\mathrm{lo}})(\omega_{\mathrm{hi}}-\omega_{\mathrm{lo}})}, & \zeta \in [\zeta_{\mathrm{lo}},\zeta_{\mathrm{hi}}] \quad \text{and} \quad \omega \in [\omega_{\mathrm{lo}},\omega_{\mathrm{hi}}] \\ 0, & \text{otherwise} \end{cases} \tag{8}$$

Gaussian:

$$f(\zeta,\omega) = \frac{1}{2\pi \det(\boldsymbol{S})} \exp\left[-\frac{(\boldsymbol{p}-\boldsymbol{p}_0)^T \boldsymbol{S}^{-1}(\boldsymbol{p}-\boldsymbol{p}_0)}{2}\right] \tag{9}$$

where $\boldsymbol{p} = [\zeta \;\; \omega]^T$, $\boldsymbol{p}_0$ is a 2×1 vector of their nominal values, and $\boldsymbol{S}$ is a 2×2 covariance matrix.

We use the performance measures (3–5) (with $t_v = 4\pi$ s) to evaluate the positive zero vibration (ZV), zero vibration and derivative (ZVD), zero vibration and double derivative (ZVDD), extra insensitive (EI), and two-hump extra insensitive (2-hump EI) shaper designs. More background on input shaping can be found in Refs. 1, 2, and 4 and the references therein. The ZV shaper is the most basic type of shaper, whereas the ZVD and EI shapers are more insensitive to parameter uncertainty, and the ZVDD and 2-hump EI are even more insensitive. The ZVD and EI shapers are of equal length (and, hence, speed), and the ZVDD and 2-hump EI are of equal length (longer than the ZVD and EI).

Figures 1 and 2 display the cost J in Eqs. (4) and (5) as a function of the size of Gaussian uncertainty (7) in ω and ζ, respectively. Here, σ is the standard deviation in the parameter ω_{model} or ζ_{model} normalized to the modeled value. From Fig. 1, for lower Gaussian uncertainty levels in ω, the ZVD and ZVDD shapers lead to lower expected residual vibration levels than the equivalent length EI and 2-hump EI shapers, whereas the EI-type shapers yield lower expected residual vibration levels at larger Gaussian uncertainties in ω. Specifically, for $\sigma < 0.15$, the ZVD shaper leads to lower levels of expected residual vibration than the equivalent length EI shaper. Similarly, for $\sigma < 0.21$, the ZVDD shaper leads to lower levels of expected residual vibration than the equivalent speed 2-hump EI shaper.

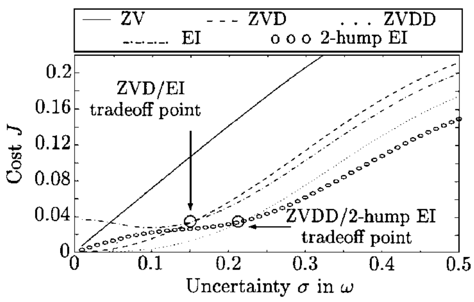

Fig. 1 Expected residual vibration vs Gaussian uncertainty levels in ω.

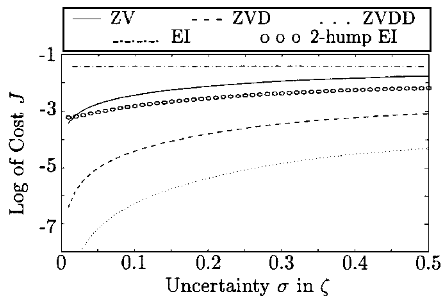

Fig. 2 Expected residual vibration vs Gaussian uncertainty levels in ζ.

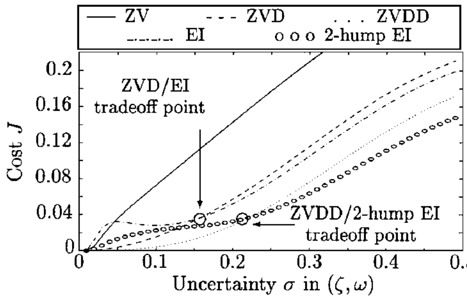

Fig. 3 Expected residual vibration vs Gaussian uncertainty levels in (ζ, ω).

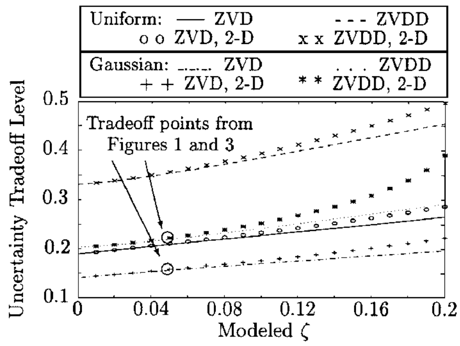

Fig. 4 Tradeoff points below which the ZVD and ZVDD methods yield lower expected residual vibration levels compared with equivalent length EI and 2-hump EI shapers, respectively.

From Fig. 2, we see that the ZVD and ZVDD yield lower expected residual vibration for all Gaussian uncertainty levels in ζ. The differences in the cost of the various shaper designs are significant, and hence, the cost is plotted on a (base 10) log scale in Fig. 2 to more clearly analyze the curves. Note that the EI method, while being more robust to frequency variations compared with the ZV shaper, leads to a larger expected residual vibration than the ZV when there is damping uncertainty.

We also evaluated the residual vibration levels as a function of the size of uniform uncertainties (6) in ω and ζ. The results show similar trends as those in Figs. 1 and 2.

Figure 3 shows the cost J in Eq. (3), where the distribution of both parameters ω and ζ is taken into account. The joint density function in Eq. (9) is used, where $f(\zeta,\omega)$ is an independent identically distributed Gaussian with the normalized standard deviation σ being the same for both ζ and ω (equal uncertainty in both parameters). We again see the trend that for smaller uncertainty levels, the ZVD and ZVDD shapers lead to lower expected residual vibration levels than the EI and 2-hump EI shapers, whereas the EI-type shapers yield lower expected residual vibration levels at larger uncertainties. Evaluations of the cost (3) with a uniform joint density function (8), where the size of uncertainty in both ω and ζ are the same, show similar trends. Specifically, as shown in Fig. 3 for $\sigma < 0.16$, the ZVD shaper leads to lower levels of expected residual vibration than the equivalent length EI shaper. Similarly, for $\sigma < 0.22$, the ZVDD shaper leads to lower levels of expected residual vibration than the equivalent length 2-hump EI shaper.

The results presented in Figs. 1–3 are for a modeled ζ of 0.05. Although the trends shown in Figs. 1–3 are similar, the expected residual vibration levels do depend on the modeled ζ, and the uncertainty levels at which the performances of the EI-type shapers surpass those of the derivative-type shapers vary with the modeled ζ. Figure 4 presents the uncertainty levels at which the performances of the EI-type shapers surpass those of the derivative-type shapers for uniform and Gaussian uncertainties in ω and in both (ω, ζ). In Fig. 4, the tradeoff points from Figs. 1 and 3 when $\zeta_{\mathrm{model}} = 0.05$ are noted. Depending on the cost function used (as labeled by the various curves), the numerical tradeoff levels in Fig. 4 represent either uniform uncertainty level or the standard deviation σ for Gaussian uncertainties. A uniform uncertainty size of 0.1 means that ω is expected to vary uniformly from $0.9\omega_{\text{model}}$ to $1.1\omega_{\text{model}}$. The curves labeled two-dimensional (2-D) represent the use of the cost J in Eq. (3) with uncertainty in both (ω, ζ). The other curves use the cost J in Eq. (4), where there is uncertainty in ω only, and there is a tradeoff level between using the derivative-type shapers relative to the EI-type shapers. When there is uncertainty in ζ, evaluations of the cost J in Eq. (5) show that the derivative-type shapers are always superior to the EI-type shapers (c.f. Fig. 2).

Hybrid Shaper Designs

The preceding section showed an analysis of several traditional input shaping designs (ZV, ZVD, EI, etc.) using a metric of expected residual vibration [cost functions (3), (4), and (5)]. Optimal shapers that minimize the expected residual vibration have been solved for and characterized.[3] For lightly damped systems, these optimal shapers yield shorter shaper lengths than those of traditional designs and, hence, lead to faster maneuvers, while yielding minimal expected residual vibration levels. It was shown that optimal shapers that minimize the expected residual vibration lead to more ZVD-like shapers for small sizes of parameter uncertainty whereas optimal designs for larger sizes of parameter uncertainty yield EI-like shapers.[3]

Based on this interpretation, it may be possible to derive shapers that yield near-minimal levels of residual vibration by interpolating between ZVD and EI shaper designs and avoid the complexity of solving the full optimization problem. In the full optimization problem, the cost function (3), (4), or (5) is to be minimized over all of the shaper parameters (the impulse amplitudes A_i and the impulse times t_i). In this section, we will derive shapers with parameters constrained to be between ZVD and EI parameters that achieve near-optimal solutions.

A hybrid shaper design is obtained as an interpolation between ZVD and EI shapers by taking a linear convex combination of ZVD and EI parameters (impulse times and amplitudes):

$$t_{\text{HYB}i} = (1-\lambda)t_{\text{ZVD}i} + \lambda t_{\text{EI}i} \tag{10}$$

$$A_{\text{HYB}i} = (1-\lambda)A_{\text{ZVD}i} + \lambda A_{\text{EI}i} \tag{11}$$

where ZVD and EI stand for their corresponding shaper parameters, i indicates the ith impulse of the shaper, and λ $(0 < \lambda < 1)$ is a design parameter chosen to obtain acceptable near-optimal solutions for the performance index (3), (4), or (5). Because there is only one variable, λ, to optimize over, the hybrid shaper designs are far easier to compute than the full optimization problem.

Because ZVD and EI designs are only functions of ζ_{model} (Ref. 4), the hybrid shaper design in Eqs. (10) and (11) will also only vary with ζ_{model}. This approach constitutes a very straightforward procedure for deriving near-optimal input shaping designs without actually solving the full optimization problem using Eqs. (3), (4), or (5), which require sophisticated and time-consuming numerical routines. By applying the procedure in Eqs. (10) and (11), we simply adjust the design parameter λ and evaluate the performance index (3), (4), or (5) until a minimal level of expected residual vibration is attained.

The flexibility in adjusting λ leads to a near-optimal shaper design. To determine the hybrid shaper design defined in Eqs. (10) and (11), we interpolate between the ZVD and EI parameters by optimizing over the design parameter λ $(0 < \lambda < 1)$. In this work, we have chosen the metric defined in Eq. (4) applied to three different uniform uncertainty ranges of normalized frequency $\omega \in [0.9, 1.1]$, $[0.8, 1.2]$, and $[0.7, 1.3]$ and Gaussian uncertainties for $\sigma =$ 0.1, 0.2, and 0.3. We set the frequency $\omega_{\text{model}} = 1$ and let the damping coefficients ζ_{model} vary from 0.01 to 0.2.

The expected residual vibration of the hybrid shapers for uniform and Gaussian uncertainties in frequency are shown in Fig. 5, and it can be seen that this design approach can achieve very near-minimal expected residual vibration levels. The hybrid shapers yield expected residual vibration levels close to optimal by suitably adjusting the design parameter λ for a given size of uncertainty in frequency (Fig. 5). In particular, the hybrid designs lead to lower levels of expected residual vibration than either ZVD or EI shaping methods.

The corresponding optimal design parameters λ are shown in Fig. 6 for uniform and Gaussian uncertainties in frequency. These values of λ yield near-optimal levels of expected residual vibration to those obtained by solving the full optimization problem[3] in Eq. (4). It is clear that for smaller sizes of uncertainty (such as $\omega \in [0.9, 1.1]$ or $\sigma = 0.1$) that the design parameter λ should be smaller, and for larger sizes of uncertainty ($\omega \in [0.7, 1.3]$ or $\sigma = 0.3$) the optimal λ is larger.

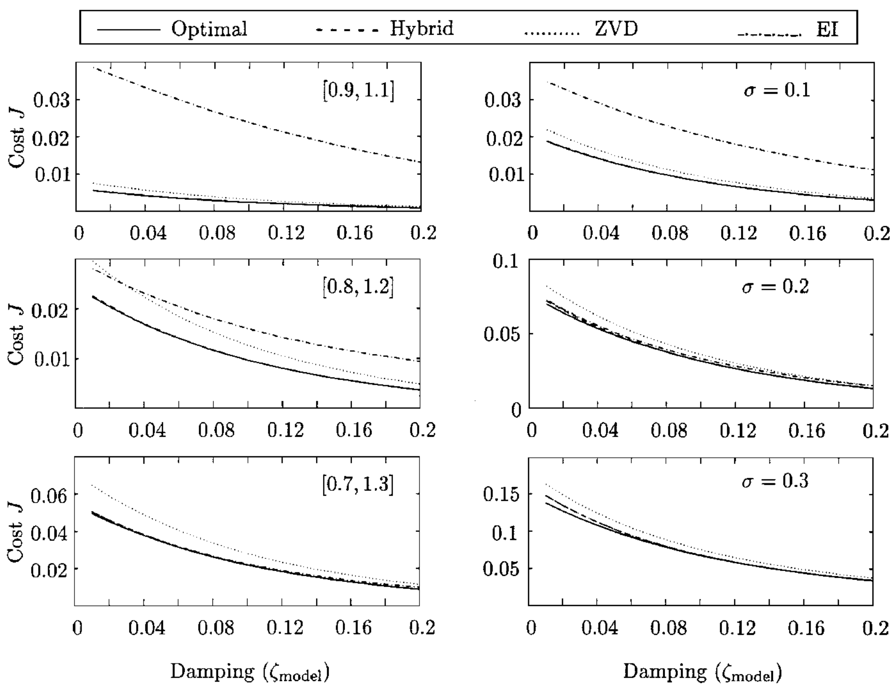

Fig. 5 Expected residual vibration for uniform uncertainty in frequency (left) and Gaussian uncertainty in frequency (right).

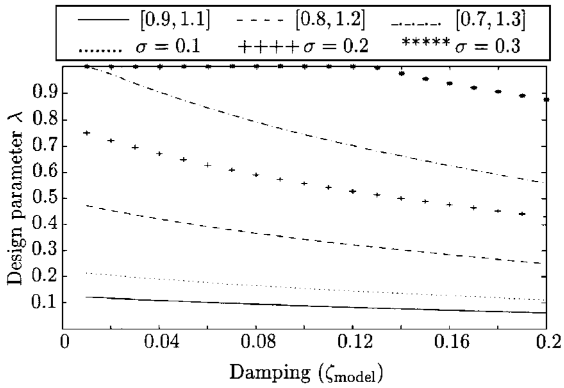

Fig. 6 Optimal design parameters for hybrid shapers under uniform and Gaussian uncertainty in frequency.

As shown in Fig. 6, for $\sigma = 0.3$ and low damping ($\zeta_{\text{model}} < 0.13$), the design parameter is fixed at the value $\lambda = 1$. It is possible to have $\lambda > 1$, but this will cause the second amplitude A_2 to be negative and we were interested in positive amplitude shapers [see Eq. (2)]. This suggests that, when uncertainties are very large (and ζ_{model} is small), the best designs or more appropriate designs are those obtained by taking convex combinations of ZVDD and 2-hump EI shapers.

Conclusions

The expected residual vibration performance measure easily incorporates the probability distribution of modeling errors in both natural frequencies and damping coefficients. Using this new measure to analyze several traditional input shaping designs shows that the EI and multihump shapers yield lower expected residual vibration than the equivalent length ZVD and derivative-type shapers for large uncertainties in the natural frequency or large uncertainties in both natural frequency and damping constant. However, the ZVD and derivative shapers lead to smaller residual vibration levels when there is uncertainty in the damping. The results presented also clearly indicate at what uncertainty levels the performance of the EI and multihump shapers become superior to the ZVD and derivative-type shapers, and these results are useful in guiding the choice of the most appropriate type of shaper given the expected variation in the system parameters.

Finally, the proposed hybrid shaper design provides a useful method for deriving shapers that can achieve near-minimal expected residual vibration levels by adjusting only a single design parameter. This method provides a straightforward procedure without resorting to sophisticated numerical optimizers. The simplicity of the hybrid design approach is valuable for an online adaptive controller, whereas the full optimal designs are more feasible in applications requiring only offline computations.

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Formalized Approach to Obtaining Optimal Coefficients for Coning Algorithms

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Introduction

ATTITUDE computation has played a key role in strapdown inertial navigation systems because the computed attitude has been continuously used for transforming the vehicle acceleration measured by the accelerometers rigidly attached to the host vehicle. Commonly used attitude updating algorithms for strapdown systems are the Euler method, the direction cosine method, and the quaternion method. Among them, the quaternion method is quite popular due to the advantages of its nonsingularity, simplicity, and computational efficiency.[1] Further improvements on the quaternion method have been made possible by Bortz,[2] Jordan,[3] Miller,[4] Ignagni,[5] Lee et al.,[6] and Jiang[7] using the concepts of rotation vector. It has been proven that the quaternion updating method with the rotation vector can effectively suppress the noncommutativity error,[4–7] which is one of the major error sources in numerical solutions of the attitude equation. The concept for optimizing coning compensation algorithms for strapdown inertial systems was first introduced and applied by Miller.[4] Ignagni[5] showed that, although only pure coning angular rate environment is considered, the algorithm optimization procedure is applicable to generalized vibrational environments.

Recently, Ignagni[8] proposed the concept leading to optimal accuracy characteristics and minimum computational throughput required in a pure coning environment. The algorithms that use the enhancement concept have the simplest form generating many distinct sensor-data cross products and optimal coefficients minimizing the coning compensation error. In this Note, a formalized approach to determining the optimal coefficients in the coning algorithms is proposed. The effectiveness of the proposed approach is demonstrated by applying not only many of the existing coning algorithms but also the determination of the optimal coefficients for six-data-interval case.

Quaternion Update Using the Rotation Vector

The key operation of the attitude algorithm is to properly update the quaternion and rotation vector.[4] The quaternion update $\bar{Q}(t+h)$ is obtained by the following quaternion multiplication:

$$\bar{Q}(t+h) = \bar{Q}(t) * \bar{q}(h) \tag{1}$$

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